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On bicyclic graphs whose second largest eigenvalue does not exceed 1 ☆

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Abstract

Connected graphs in which the number of edges equals the number of vertices plus one are called bicyclic graphs. In this paper, all bicyclic graphs whose second largest eigenvalue does not exceed 1 have been determined.

AMS classification: 05C50

Keywords: Bicyclic graphs; Eigenvalue; Induced subgraph

1. Introduction

The graphs in this paper are simple. Let $A(G)$ be a (0, 1)-adjacency matrix of G. Since $A(G)$ is symmetric, its eigenvalues are real. Without loss of generality, we can write them as $\lambda_1(G) \geqslant \lambda_2(G) \geqslant \cdots \geqslant \lambda_n(G)$ and call them the eigenvalues of G. The second largest eigenvalue of a graph is closely related to its diameter, and the diameter is very important for a network. Therefore it is of great practical

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value to study the second largest eigenvalue of graphs. In [2], Cvetković asked if it was possible to determine all the graphs whose second largest eigenvalue λ_2 does not exceed 1. In subsequent years, some results concerning this problem have been obtained (see [1,7]). In 1989, Hong [8] determined all the trees with $\lambda_2 < 1$. In 1998, Shu [9] determined all the trees with $\lambda_2 = 1$. In 2004, Xu [10] determined all the unicyclic graphs with $\lambda_2 \leqslant 1$.

Connected graphs in which the number of edges equals the number of vertices plus one are called bicyclic graphs. In this paper, we will discuss the second largest eigenvalue of bicyclic graphs. Our main result is that a bicyclic graph G satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the following graphs G_i $(i = 1, 2, \ldots, 14)$, where $s \geqslant 0$, $t \geqslant 0$.

2. Preliminaries

Denote by C_n and P_n the cycle and the path, respectively, each on n vertices. Let $G - x$ denote the graph that arises from G by deleting the vertex $x \in V(G)$. Let C_p and C_q be two vertex-disjoint cycles. Suppose that v_1 is a vertex of C_p and v_l is a vertex of C_q. Joining v_1 and v_l by a path $v_1v_2\cdots v_l$ of length $l-1$, where $l \geqslant 1$ and $l = 1$ means identifying v_1 with v_l, the resulting graph (Fig. 2), denoted by $B(p, l, q)$, is called an ∞-graph. Without loss of generality, we may assume that $p \leqslant q$. Let P_{l+1}, P_{p+1} and P_{q+1} be three vertex-disjoint paths, where $1 \leqslant l \leqslant p \leqslant q$ and at most one of them is 1. Identifying the three initial vertices and terminal vertices of them, respectively, the resulting graph (Fig. 3), denoted by

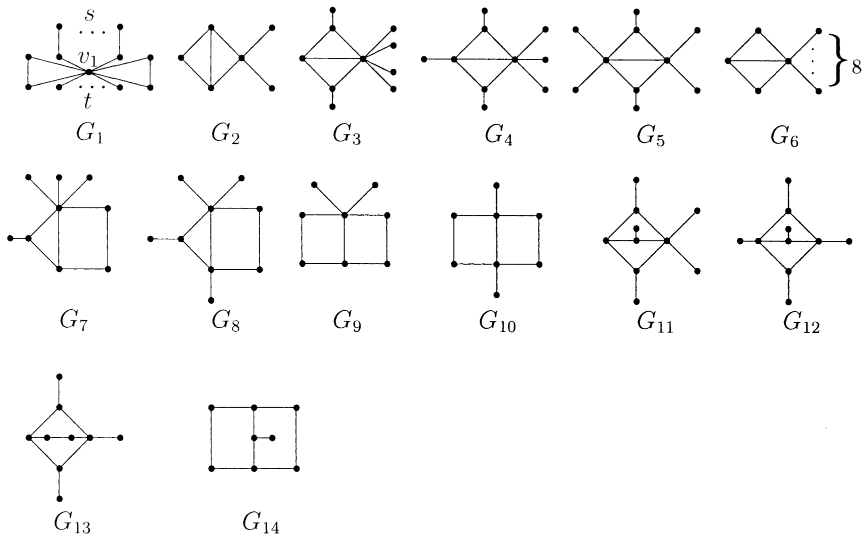

Fig. 1. $G_1 - G_{14}$.

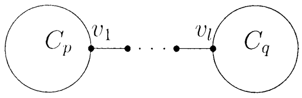

Fig. 2. $B(p, l, q)$.

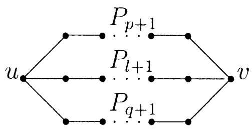

Fig. 3. $\theta(p, l, q)$.

$\theta(p, l, q)$, is called a θ-graph. Then bicyclic graphs consist of two types of graphs: one type, denoted by $\mathscr{B}_\infty$, are those graphs each of which is an ∞-graph with trees attached; the other type, denoted by $\mathscr{B}_\theta$, are those graphs each of which is a θ-graph with trees attached. We will use $\mathscr{B}(p, l, q)$ to denote the set of all bicyclic graphs which are $B(p, l, q)$ with trees attached, and $\Theta(p, l, q)$ to denote the set of all bicyclic graphs which are $\theta(p, l, q)$ with trees attached.

In order to complete the proof of our main result, we need the following lemmas.

Lemma 1 [4]. *Let V' be a subset of vertices of a graph G, $|V(G)| = n$ and $|V'| = k$, then*

$$\lambda_i(G) \geqslant \lambda_i(G - V') \geqslant \lambda_{i+k}(G), \quad (1 \leqslant i \leqslant n - k).$$

Lemma 2 [3–5, 10]. *Let u be a vertex of G, $N(u)$ be the set of all vertices adjacent to u and $C(u)$ be the set of all cycles containing u. The characteristic polynomial of G satisfies*

$$P(G; \lambda) = \lambda P(G - u; \lambda) - \sum_{v \in N(u)} P(G - u - v; \lambda) - 2 \sum_{Z \in C(u)} P(G \backslash V(Z); \lambda).$$

Lemma 3 [4]. *The spectrum of a cycle C_n consists of the numbers $2\cos(2\pi/n)i$ $(i = 1, \ldots, n)$, and the spectrum of the path P_n consists of the numbers $2\cos[2\pi/(n + 1)]i$ $(i = 1, \ldots, n)$.*

Lemma 4. *Let G_i $(i = 1, \ldots, 14)$ be the bicyclic graphs as given in Fig. 1. Then $\lambda_2(G_i) = 1$.*

Proof. From the tables of spectra of connected graphs with n vertices for $4 \leqslant n \leqslant 7$ in [3–6] and the straightforward calculation via the MATLAB Programming, we can easily see that

$$\lambda_2(G_i) = 1, \quad i = 2, \ldots, 14.$$

For G_1, applying Lemma 2 to the vertex v_1, we have

$$P(G_1;\lambda) = \lambda^{t-1}(\lambda^2-1)^{s+1}[\lambda^4-(s+t+5)\lambda^2-4\lambda+t].$$

Write

$$g(\lambda) = \lambda^4-(s+t+5)\lambda^2-4\lambda+t.$$

Then $g(0)=t \geqslant 0$, $g(1)=-s-8<0$, $g(-1)=-s \leqslant 0$. It follows that $g(\lambda)=0$ has only one root greater than 1. Hence $\lambda_2(G_1)=1$. This completes the proof. □

Lemma 5. *Let B_i $(i=1,\dots,30)$ be the graphs as given in Fig. 4 below. Then* $\lambda_2(B_i)>1$.

Proof. From the tables of spectra of connected graphs with n vertices for $6 \leqslant n \leqslant 7$ in [3–5] and the straightforward calculation via the MATLAB Programming, we can easily obtain that

$$\lambda_2(B_1)=1.056;\quad \lambda_2(B_2)=1.210;\quad \lambda_2(B_3)=1.18798;$$
$$\lambda_2(B_4)=1.06166;\quad \lambda_2(B_5)=1.22013;\quad \lambda_2(B_6)=1.0662;$$
$$\lambda_2(B_7)=1.0495;\quad \lambda_2(B_8)=1.1464;\quad \lambda_2(B_9)=1.0354;$$
$$\lambda_2(B_{10})=1.126;\quad \lambda_2(B_{11})=1.29201;\quad \lambda_2(B_{12})=1.21076;$$
$$\lambda_2(B_{13})=1.082;\quad \lambda_2(B_{14})=1.0887;\quad \lambda_2(B_{15})=1.1428;$$
$$\lambda_2(B_{16})=1.1701;\quad \lambda_2(B_{17})=1.25235;\quad \lambda_2(B_{18})=1.1395;$$
$$\lambda_2(B_{19})=1.1776;\quad \lambda_2(B_{20})=1.08239;\quad \lambda_2(B_{21})=1.0705;$$
$$\lambda_2(B_{22})=1.2077;\quad \lambda_2(B_{23})=1.26848;\quad \lambda_2(B_{24})=1.18994;$$
$$\lambda_2(B_{25})=1.24698;\quad \lambda_2(B_{26})=1.732;\quad \lambda_2(B_{27})=1.360;$$
$$\lambda_2(B_{28})=1.264;\quad \lambda_2(B_{29})=1.229;\quad \lambda_2(B_{30})=1.138.$$

These imply that $\lambda_2(B_i)>1$ for $i=1,\dots,30$. The proof is completed. □

Lemma 6. *A graph G in $\Theta(2,1,2)$ satisfies $\lambda_2(G)\leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i=2,\dots,6)$ as given in Fig. 1.*

Proof. Let $G \in \Theta(2,1,2)$ such that $\lambda_2(G)\leqslant 1$. If G has an induced subgraph isomorphic to one of B_i $(i=1,2)$ as given in Fig. 4, then by Lemmas 1 and 5, we have $\lambda_2(G)\geqslant\lambda_2(B_i)>1$, a contradiction. So G must have the form $\theta(2,1,2)[r_1,r_2,r_3,r_4]$ as given in Fig. 4. By symmetry of $\theta(2,1,2)$, we may assume that $r_1\geqslant r_3$ and $r_2\geqslant r_4$. If G has an induced subgraph isomorphic to one of B_i $(i=3,4,5)$ as given in Fig. 4, then by Lemmas 1 and 5, we have $\lambda_2(G)\geqslant\lambda_2(B_i)>1$, a contra-

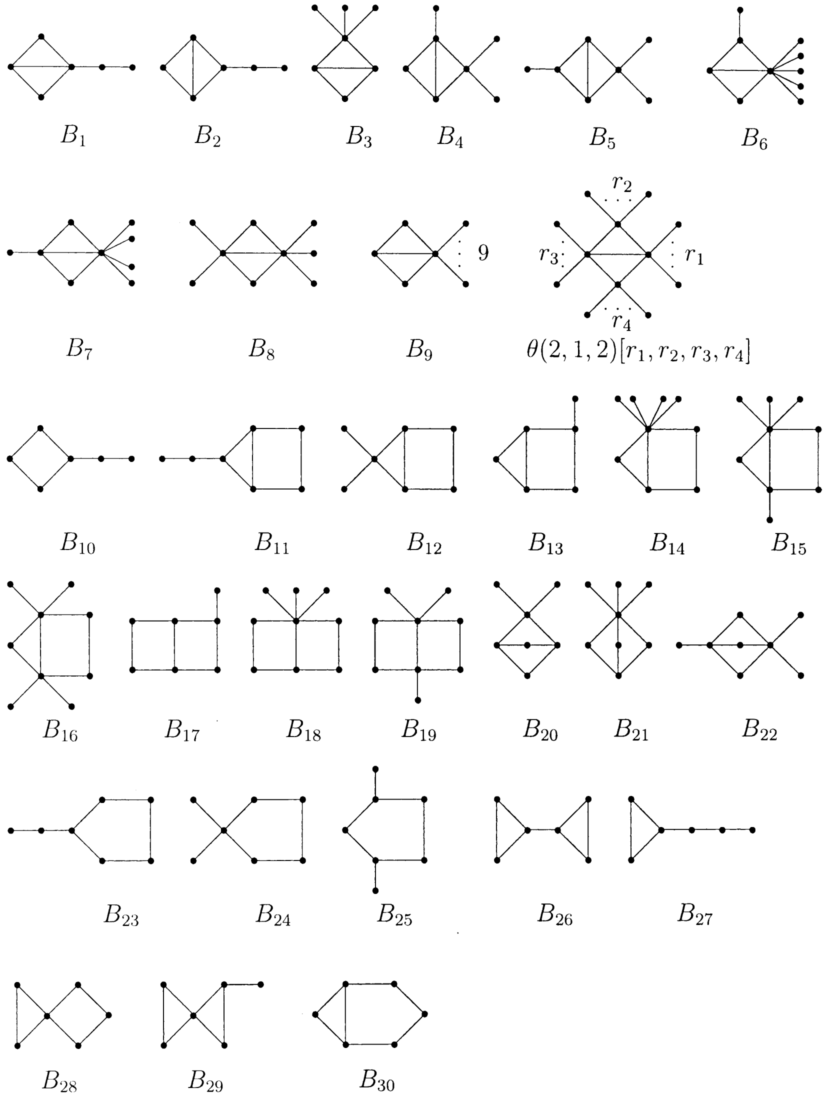

Fig. 4. $B_1 - B_{30}, \theta(2, 1, 2)[r_1, r_2, r_3, r_4]$.

diction. So $r_2 \leqslant 2$, and r_1, r_3 and r_4 must be all equal to 0 when $r_2 = 2$. If G has an induced subgraph isomorphic to one of B_i $(i = 6, 7, 8, 9)$ as given in Fig. 4, then by Lemmas 1 and 5, we have $\lambda_2(G) \geqslant \lambda_2(B_i) > 1$, a contradiction. So G can only be the graphs G_i $(i = 2, \ldots, 6)$ as given in Fig. 1 or their induced bicyclic subgraphs. For G_i $(i = 2, \ldots, 6)$, we have known in Lemma 4 that $\lambda_2(G_i) = 1$. This completes the proof. $\square$

Lemma 7. *A graph G in $\Theta(2, 1, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 7, 8)$ as given in Fig. 1.*

Proof. Let $G \in \Theta(2, 1, 3)$ such that $\lambda_2(G) \leqslant 1$. If G has an induced subgraph isomorphic to one of B_i $(i = 10, 11)$ as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) > 1$, a contradiction. So G must have the form $\theta(2, 1, 3)^*$ which is the $\theta(2, 1, 3)$ with some pendent edges attached. If G has an induced subgraph isomorphic to one of B_i $(i = 12, \ldots, 16)$ as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) > 1$, a contradiction. So G can only be the graphs G_i $(i = 7, 8)$ as given in Fig. 1 or their induced bicyclic subgraphs. Moreover, by Lemma 4 we have that $\lambda_2(G_7) = 1$, $\lambda_2(G_8) = 1$. This completes the proof. $\square$

Lemma 8. *A graph G in $\Theta(3, 1, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of the graphs G_i $(i = 9, 10)$ as given in Fig. 1.*

Proof. Let $G \in \Theta(3, 1, 3)$ such that $\lambda_2(G) \leqslant 1$. If G has an induced subgraph isomorphic to one of B_i $(i = 17, 18, 19)$ as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) > 1$, a contradiction. So G can only be the graphs G_i $(i = 9, 10)$ as given in Fig. 1 or their induced bicyclic subgraphs. Moreover, by Lemma 4 we have that $\lambda_2(G_9) = 1$, $\lambda_2(G_{10}) = 1$. This completes the proof. $\square$

Lemma 9. *A graph G in $\Theta(2, 2, 2)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 11, 12)$ as given in Fig. 1.*

Proof. Let $G \in \Theta(2, 2, 2)$ such that $\lambda_2(G) \leqslant 1$. If G has an induced subgraph isomorphic to one of B_i $(i = 10, 20, 21, 22)$ as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) > 1$, a contradiction. So G can only be the graphs G_i $(i = 11, 12)$ as given in Fig. 1 or their induced bicyclic subgraphs. Moreover, by Lemma 4 we have that $\lambda_2(G_{11}) = 1$, $\lambda_2(G_{12}) = 1$. This completes the proof. $\square$

Lemma 10. *A graph G in $\Theta(2, 2, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of the graph G_{13} as given in Fig. 1.*

Proof. Let $G \in \Theta(2, 2, 3)$ such that $\lambda_2(G) \leqslant 1$. If G has an induced subgraph isomorphic to one of B_i $(i = 23, 24, 10, 25)$ as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) > 1$, a contradiction. So G can only be the graph G_{13} as given in Fig. 1 or induced bicyclic subgraphs of G_{13}. Moreover, by Lemma 4 we have that $\lambda_2(G_{13}) = 1$. This completes the proof. $\square$

Lemma 11. *A graph G in $\Theta(3, 2, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of the graph G_{14} as given in Fig. 1.*

Proof. Let $G \in \Theta(3, 2, 3)$ such that $\lambda_2(G) \leqslant 1$. If G has an induced subgraph isomorphic to one of B_i $(i = 23, 24)$ as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) > 1$, a contradiction. So G can only be the graph G_{14} as given in Fig. 1 or induced bicyclic subgraphs of G_{14}. Moreover, by Lemma 4 we have that $\lambda_2(G_{14}) = 1$. This completes the proof. □

3. Main results

Theorem 1. *A bicyclic graph G satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 1, \ldots, 14)$ as given in Fig. 1, where $s \geqslant 0$, $t \geqslant 0$.*

The proof of Theorem 1 follows immediately from Theorems 2 and 3.

Theorem 2. *Let $G \in \mathscr{B}_\infty$. Then $\lambda_2(G) \geqslant 1$, and the equality holds if and only if G is the graph G_1 as given in Fig.1, where $s \geqslant 0$, $t \geqslant 0$.*

Proof. For $G \in \mathscr{B}_\infty$, let $B(p, l, q)$ be the ∞-graph in G, and $v_1 v_2 \cdots v_l$ be the path joining the cycles C_p and C_q in $B(p, l, q)$. We consider the following three cases.

Case 1. $l > 2$. Then there exist two components of $G - v_2$, denoted by U_1 and U_2 respectively, such that both U_1 and U_2 are unicyclic graphs. By Lemmas 1 and 3, we have $\lambda_1(U_1) \geqslant 2$, $\lambda_1(U_2) \geqslant 2$. Therefore

$$\lambda_2(G) \geqslant \min\{\lambda_1(U_1), \lambda_1(U_2)\} \geqslant 2.$$

Case 2. $l = 2$.

Subcase 2.1. $q \geqslant 4$. Then there exist two components of $G - v_2$, denoted by U_3 and T_1 respectively, such that U_3 is a unicyclic graph and T_1 is a tree with $|V(T_1)| \geqslant 3$. By Lemmas 1 and 3, we have $\lambda_1(U_3) \geqslant 2$, $\lambda_1(T_1) > 1$. Therefore

$$\lambda_2(G) \geqslant \min\{\lambda_1(G_1), \lambda_1(T_1)\} > 1.$$

Subcase 2.2. $q = 3$. Then G has an induced subgraph as B_{26} given in Fig. 4. By Lemmas 1 and 5, we have

$$\lambda_2(G) \geqslant \lambda_2(B_{26}) > 1.$$

Case 3. $l = 1$.

Subcase 3.1. $p \geqslant 4$ and $q \geqslant 4$. Then all components of $G - v_1$ are trees, and there exist two components, denoted by T_2 and T_3 respectively, such that $|V(T_2)| \geqslant 3$ and $|V(T_3)| \geqslant 3$. By Lemmas 1 and 3, we have

$$\lambda_2(G) \geqslant \min\{\lambda_1(T_2), \lambda_1(T_3)\} > 1.$$

Subcase 3.2. $p = 3$ and $q > 4$. Then G has an induced subgraph as B_{27} given in Fig. 4. By Lemmas 1 and 5, we have

$$\lambda_2(G) \geqslant \lambda_2(B_{27}) > 1.$$

Subcase 3.3. $p = 3$ and $q = 4$. Then G has an induced subgraph as B_{28} given in Fig. 4. By Lemmas 1 and 5, we have

$$\lambda_2(G) \geqslant \lambda_2(B_{28}) > 1.$$

Subcase 3.4. $p = 3$ and $q = 3$. If G has an induced subgraph as B_{27} or B_{29} given in Fig. 4, then by Lemmas 5 and 1 we have $\lambda_2(B_{27}) > 1$, $\lambda_2(B_{29}) > 1$ and $\lambda_2(G) > 1$. Otherwise, G can only be the graphs G_1 as given in Fig. 1, where $s \geqslant 0$, $t \geqslant 0$. By Lemma 4 we have $\lambda_2(G_1) = 1$. This completes the proof. □

Theorem 3. *A bicyclic graph G in $\mathscr{B}_\theta$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 2, \ldots, 14)$ as given in Fig.* 1.

Proof. Let $G \in \mathscr{B}_\theta$ such that $\lambda_2(G) \leq 1$, and let $\theta(p, l, q)$ be the θ-graph in G, where $1 \leqslant l \leqslant p \leqslant q$. Then $G \in \Theta(p, l, q)$. We consider the following three cases.

Case 1. $l = 1$. Then $2 \leqslant p \leqslant q$.

Subcase 1.1. $p = 2$. If $q \geqslant 5$, then G must have an induced subgraph that is isomorphic to B_{27} as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) \geqslant \lambda_2(B_{27}) > 1$, a contradiction.

If $q = 4$, then G must have an induced subgraph that is isomorphic to B_{30} as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) \geqslant \lambda_2(B_{30}) > 1$, a contradiction.

For $q = 3$, in Lemma 7 we have known that a graph G in $\Theta(2, 1, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 7, 8)$ as given in Fig. 1.

For $q = 2$, in Lemma 6 we have known that a graph G in $\Theta(2, 1, 2)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 2, \ldots, 6)$ as given in Fig. 1.

Subcase 1.2. $p = 3$. If $q \geqslant 4$, then G must have an induced subgraph that is isomorphic to B_{10} as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) \geqslant \lambda_2(B_{10}) > 1$, a contradiction.

For $q = 3$, in Lemma 8 we have known that a graph G in $\Theta(3, 1, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of the graphs G_i $(i = 9, 10)$ as given in Fig. 1.

Subcase 1.3. $p \geqslant 4$. Then $q \geqslant 4$ and G must have an induced subgraph that is isomorphic to P_7. From the table of spectra of connected graphs with seven vertices in [3], we see that $\lambda_2(P_7) = 1.41421$. By Lemma 1, we have $\lambda_2(G) \geqslant \lambda_2(P_7) > 1$, a contradiction.

Case 2. $l = 2$. Then $2 \leqslant p \leqslant q$.

Subcase 2.1. $p = 2$. If $q \geqslant 4$, then G must have an induced subgraph that is isomorphic to B_{10} as given in Fig. 4. By Lemmas 1 and 5, we have $\lambda_2(G) \geqslant \lambda_2(B_{10}) > 1$, a contradiction.

For $q = 3$, in Lemma 10 we have known that a graph G in $\Theta(2, 2, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of the graph G_{13} as given in Fig. 1.

For $q = 2$, in Lemma 9 we have known that a graph G in $\Theta(2, 2, 2)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 11, 12)$ as given in Fig. 1.

Subcase 2.2. $p = 3$. If $q \geqslant 4$, then G must have an induced subgraph that is isomorphic to C_k $(k \geqslant 7)$. By Lemma 3, we see that $\lambda_2(C_k) = 2\cos\frac{2\pi}{k} > 1$ when $k \geqslant 7$. Therefore, we have $\lambda_2(G) \geqslant \lambda_2(C_k) > 1$, a contradiction.

For $q = 3$, in Lemma 11 we have known that a graph G in $\Theta(3, 2, 3)$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of the graphs G_{14} as given in Fig. 1.

Subcase 2.3. $p \geqslant 4$. Then $q \geqslant 4$ and G must have an induced subgraph that is isomorphic to C_k $(k \geqslant 8)$. By Lemma 3, we see that $\lambda_2(C_k) > 1$ when $k \geqslant 8$. Therefore, we have $\lambda_2(G) \geqslant \lambda_2(C_k) > 1$, a contradiction.

Case 3. $l \geqslant 3$. Then $q \geqslant p \geqslant 3$, and G must have an induced subgraph that is isomorphic to C_6^+ or C_k $(k \geqslant 7)$, where C_6^+ is the cycle C_6 with a pendent edge attached. From the table of spectra of connected graphs with seven vertices in [3], we see that $\lambda_2(C_6^+) = 1.25928$. By Lemma 3, we see that $\lambda_2(C_k) > 1$ when $k \geqslant 7$. Therefore by Lemma 1 we have

$$\lambda_2(G) \geqslant \lambda_2(C_6^+) > 1 \quad \text{or} \quad \lambda_2(G) \geqslant \lambda_2(C_k) > 1,$$

a contradiction.

Combining the above arguments, we have that a bicyclic graph G in $\mathscr{B}_\theta$ satisfies $\lambda_2(G) \leqslant 1$ if and only if G is an induced bicyclic subgraph of one of the graphs G_i $(i = 2, \ldots, 14)$ as given in Fig. 1. This completes the proof. □

From Theorem 1, we can see that if $G \in \mathscr{B}_\theta$ with $|V(G)| > 12$, then $\lambda_2(G) > 1$. So we have following two corollaries.

Corollary 1. *Let G be a bicyclic graph with vertex number $n > 12$. Then $\lambda_2(G) \geqslant 1$, and the equality holds if and only if G is the graph G_1 as given in Fig.* 1, *where $s \geqslant 0$, $t \geqslant 0$.*

Corollary 2. *Let G be a bicyclic graph with vertex number $n > 12$ and edge independence number q. Then $\lambda_2(G) \geqslant 1$, and the equality holds if and only if G is the graph G_1 as given in Fig.* 1, *where $s = q - 3$, $t \geqslant 1$, or $s = q - 2$, $t = 0$.*

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